

# UNIFORM ASYMPTOTIC DESCRIPTION OF THE MAIN SIGNAL

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Abstract Submission Form

2006 National Radio Science Meeting

Abstract: cartwright27052

Date Received: September 9, 2005

The determination of the dynamical evolution of a signal in a dispersive medium or system is a problem of central importance in the area of electromagnetic pulse propagation. Consider the following canonical problem in the theory of ultrawideband pulse propagation through a temporally dispersive, absorptive material: Let the half-space  $z < 0$  be vacuum, while the half-space  $z \geq 0$  is occupied by a dielectric material whose temporal frequency dispersion is described by the causal, single-resonance Lorentz model (H. A. Lorentz, *The Theory of Electrons*, 1906). Let the wave field traveling in the positive  $z$ -direction on the plane  $z = 0^+$  be a unit step function modulated sinusoidal plane wave of fixed carrier frequency  $\omega_c > 0$  with turn-on time at  $t = 0$ . With this initial field, determine the temporal evolution of the propagated signal on any given plane  $z > 0$ .

The exact, formal solution to this problem is given by the Fourier-Laplace integral representation

$$E(z, t) = \frac{1}{2\pi} \int_{ia-\infty}^{ia+\infty} \tilde{E}(0, \omega) \exp[i(\tilde{k}(\omega)z - \omega t)] d\omega, \quad (1)$$

for  $z \geq 0$  and  $a > 0$ . Here  $\tilde{E}(0, \omega) = i/(\omega - \omega_c)$  is the Fourier transform of the initial electric field component  $E(0, t) = u(t) \sin(\omega_c t)$ ,  $u(t)$  is the unit step function, and  $\tilde{k}(\omega) = (\omega/c)n(\omega)$  is the *complex wave number* in the dispersive dielectric, where  $c$  denotes the speed of light in vacuum. For a single-resonance Lorentz model dielectric, the *complex index of refraction* is given by

$$n(\omega) = \left( 1 - \frac{\omega_p^2}{\omega^2 - \omega_0^2 + 2i\delta\omega} \right)^{1/2}, \quad (2)$$

with *plasma frequency*  $\omega_p$ , *undamped resonance frequency*  $\omega_0$ , and *phenomenological damping constant*  $\delta$ . For an ultrawideband wave form whose spectrum spans a wide range of frequencies over which  $\tilde{k}$  may vary in a complicated manner, the group velocity approximation is not a valid approximation for propagation distances greater than one absorption depth into the dispersive material.

Because of the exponential appearing in the integrand of (1), the Fourier-Laplace representation is ideally suited for analysis by asymptotic expansion techniques that are valid as  $z \rightarrow \infty$ , as originally done by Brillouin (*Wave Propagation and Group Velocity*, 1960) using the method of steepest descents. An improved asymptotic approximation was later provided by Oughstun and Sherman (*Electromagnetic Pulse Propagation in Causal Dielectrics*, 1994) with the use of modern, uniform asymptotic techniques. However, the approximation of Oughstun and Sherman showed discontinuities in the propagated field and gave incorrect approximations for the pole contribution when the value of the carrier frequency of the input pulse lies within or above the absorption band of the material. Here, we provide a completely uniform asymptotic approximation to this problem. This refined asymptotic description results in a continuous evolution of the propagated field for all space-time points.

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